# **EFFECT OF ADAPTATION CONSTANT** *μ* **IN THE ACHIEVABLE AMPLIFICATION OF A REAL TIME LMS BASED ADAPTIVE FEEDBACK CANCELLER FOR PUBLIC ADDRESS SYSTEM**

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**Abstract:** This study considers the problem experienced with public address system in an area constrained environment. In an environment wherein public address system is used to address the general public, acoustic feedback occurs unpredictably which significantly degrades the acoustic quality of the information signal. An adaptive filter is then presented, with the objective to reduce the effect of acoustic feedback of the public address system that might occur at any point within the area. The system covers a 20th order multi-tapped finite impulse response (FIR) adaptive filter that is implemented in field programmable gate array (FPGA). The performance of the adaptive filter with least mean square (LMS) adaptive algorithm was presented and the effects of the adaptation constant  $\mu$  were investigated. The system was evaluated with a prepositioned speaker and microphone to which an acoustic feedback would occur by slowly increasing the gain of the audio power amplifier. An acoustic signal was also applied to a microphone as part of the evaluation. During an evaluation, the acoustic activity of the environment was observed by a spectrum analyzer and spectrograph. Results show that with large *μ*, the adaptive filter was able to quickly reduce the howling signal before the system exhibit an unstable behavior. However, results also show that the magnitude of the output information signal of the public address system with large  $\mu$  is lesser compared to a smaller  $\mu$  of the adaptive filter. This shows that the adaptation constant  $\mu$  does not only eliminate the acoustic feedback but also reduces the amplification of the public address system with acoustic feedback controller.

**Keywords:** *acoustic feedback control, acoustic feedback canceller, howling, public address, adaptive filter, least mean square, adaptation constant*

# **1. INTRODUCTION**

Adaptive filtering techniques had been applied to some systems and for the past decade, many papers had been published in the field of adaptive control and signal processing engineering that utilize the concept of adaptive filtering (Waterschoot & Moonen, 2011; Gil-Cacho, van Waterschoot, Moonen, & Jensen, 2009). The adaptive control and filtering techniques had been successfully implemented to provide solutions to some control engineering related problems (Ma, Gran, Jacobsen, & Agerkvist, 2011; Tendero-Reas & Reas, 2016; Ge & Zhang, 2003). In this study, adaptive filter will be used to control acoustic feedback problem present in public address system.

In communication, the sound quality is of major importance for being able to properly perceive and interpret the correct sound signal. One way to effectively transmit a sound signal over a distance is to use public address system. Public address system has been in use from the past in orchestras, theatres, cinemas, conference room and many more. It has greatly improved since its first use and keeps on improving from time to time. However, its efficiency has been severely affected because of some problem. One

of the problems that plagued public address system is howling. Howling is the undesirable sound that occurs when the loudspeaker signal is being picked up by the microphone and amplified by the power amplifier (Waterschoot & Moonen, 2011). In this way, a special kind of positive feedback having a closed signal loop is created. The resulting effect affects the system's performance, deteriorates the sound quality and limits the achievable amplification (Bispo  $\&$  Freitas, 2015). In literature, howling occurs when a closed loop system satisfies the two conditions of the Nyquist criterion for instability. The Nyquist criterion for instability stated that a system is unstable 1) if the product of electroacoustic forward path and the acoustic feedback path is greater than one and 2) the overall phase angle of the electroacoustic forward path and the acoustic feedback path is an integer multiple of  $2\pi$ .

Elimination of howling in public address system can be done in manual or automatic. In manual acoustic feedback control, a skilled technician is required in order to effectively eliminate howling. However, sound technicians are not always available especially in a small environment wherein the sound reinforcement is necessary. Often, the lack of availability of a skilled sound technician will lead an unskilled personnel to attempt to eliminate the howling by simply reducing the gain. Thus, sound signal will not be sufficiently amplified. In addition, the talker's movement in a room might be also constrained to a small area in order to prevent the existence of the acoustic feedback path. As a result, the talker may unexpectedly move from one place to the other in order to efficiently deliver the information to the listeners.

For the past five decades, a lot of solutions have been proposed with the objective to automatically eliminate or reduce the acoustic feedback (Waterschoot & Moonen, 2011). The process of solving the acoustic feedback problem by completely eliminating or reducing is referred as acoustic feedback control. However, technicians still prefer to eliminate or reduce acoustic feedback in a manual manner. The main reason why acoustic technicians prefer manual acoustic feedback control is reliability issues. Acoustic feedback control techniques are categorized into four, 1) Phase Modulation (PM), 2) gain reduction, 3) spatial filtering and 4) room modeling. In general, the objective of acoustic feedback control is to prevent the system that will satisfy the two conditions of the Nyquist criterion for instability. In phase modulation method, the acoustic feedback controller manipulates the overall phase of the closed loop so that it will not satisfy the phase condition of the Nyquist criterion for instability. While in the gain reduction method, it simply reduces the gain of the acoustic forward path in order to prevent the closed loop to satisfy the magnitude condition of the Nyquist criterion for instability. On the other hand, the spatial filter uses a microphone array to manipulate the beam of the microphone array and focused it to the talker, while the null is directed towards the loudspeaker. The objective in spatial filtering is to avoid the existence of acoustic feedback path, hence, it is guaranteed that closed loop will not satisfy the two conditions of the Nyquist criterion for instability. Lastly, room modeling technique models the acoustic environment or the acoustic feedback path and the resulting model is subtracted to the signal from the microphone. The result is a feedback compensated electroacoustic forward path. An adaptive filter with an adaptive algorithm is being used to model the acoustic feedback path. The adaptive algorithm manipulates the coefficients of the adaptive filter to obtain an equivalent model of the acoustic feedback path.

#### *The Acoustic Feedback and Acoustic Feedback Canceller*

A typical block diagram of a simple public address system is shown in Figure 1. It is composed of a microphone, an amplifier or an audio processing circuit and a loudspeaker. The microphone picks the acoustic signal  $v(z)$  and then convert it to electrical signal  $d(z)$ . The electroacoustic signal  $d(z)$  is then amplified and enhanced by an audio processing circuit. The amplified and enhanced electroacoustic signal is then converted back to acoustic signal through a loudspeaker. Usually, the microphone and the loudspeaker are positioned in such a way that the loudspeaker sound does not hit the microphone. However, in some cases such as an area constrained environment, part of the acoustic signal from the loudspeaker is being fed back to the microphone through the acoustic feedback path. The acoustic feedback path is referred to the environment wherein the acoustic signals from the loudspeaker return to the microphone. In this case, the acoustic signal from the loudspeaker may unavoidably be reflected by the boundaries of walls, floors or ceilings. Acoustic feedback path couples the loudspeaker and the microphone and as a result, a closed loop system was established (Bispo & Freitas, 2015). The closed loop system is shown in Figure 1, which greatly affect the performance of the public address system (Waterschoot & Moonen, 2011). Among the different artifacts that are produced by the acoustic coupling between the loudspeaker and the microphone, howling effect is the most characteristic one. The transfer function of the public address system, in consideration to the existence of the acoustic feedback path, is shown in Eq. 1. In Eq. (1), the  $G(z)$  and  $F(z)$  are the mathematical models of the acoustic forward path and acoustic feedback path, respectively (Waterschoot & Moonen, 2011). The acoustic feedback path model  $F(z)$  is linear, time varying and of finite order. The linearity assumption is generally considered to be a reasonable one since the effects of sound propagation and reflections in the acoustic environment are quasi-level and independent. Meanwhile, the product of  $G(z)$  and  $F(z)$  in the denominator in Eq. (1) is often referred to as the loop response of the system.

$$
\frac{d(z)}{v(z)} = \frac{G(z)}{1 - G(z)F(z)}\tag{1}
$$



Figure 1. Typical block diagram of a public address system.

[51]

The loop response of a closed loop system plays an important role in the overall performance of the public address system. It is known that a closed loop system may exhibit instability, which may lead to oscillation that in an acoustic system is perceived as howling. In control system, a closed loop system will exhibit instability if the two conditions of Nyquist criterion for instability are satisfied. The Nyquist criterion for instability is shown below for magnitude and phase condition, respectively.

$$
|G(z)F(z)| \ge 1,\tag{2}
$$

$$
\angle G(z)F(z) = n(2\pi), \ \ n = \cdots, -2, -1, 0, 1, 2, \cdots \tag{3}
$$

Eq. (2) explains that for any acoustic signal of any quantity will gradually increase from time to time as it is being fed back because the gain, which is the magnitude of the product of  $G(z)$  and  $F(z)$ , is greater than one. Furthermore, the gradual increase in acoustic signal as it is being fed back can be also attributed to an in-phase relationship of the acoustic input and output signal. Eq. (3) shows that the acoustic input and output aids one another if the acoustic output signal is being fed back. Hence, the acoustic signal will gradually increase from time to time (Gou, 2012). Magnitude condition in the Nyquist criterion for instability, if fulfilled, will cause the signal traveling around the loop to further increase and the phase condition in the Nyquist criterion for instability, if fulfilled, will cause the signal to add up in phase to the microphone signal.

The magnitude and phase conditions in the Nyquist criterion for instability shown in Eq. (2) and Eq. (3) are very important in the design of an effective acoustic feedback control. Acoustic feedback control method will be designed with the objective to prevent either one of the conditions in Eq. (2) and Eq. (3) from being met.

Proper sound design can help alleviate howling (Berdahl, Widrow, & Flores, 2005). Often the speaker is placed in front of the microphone. Thus, this configuration can significantly reduce the occurrence of howling since both the microphone and the loudspeaker are directionally dependent. The implementation of this scheme will require a skilled sound technician. Also, the sound technician will often test the system by increasing the gain until feedback occurs and then a notch filter is applied corresponding to the peaks where howling occurs in the loop transfer function. In addition, the implementation of this method in some situation would require too many notch filters (Berdahl *et al*., 2005). Furthermore, the loop gain of the public address system in some circumstances of this technique is required to be decreased. Hence, the microphone will be placed closer to the talker. In this sense, the view on the talker will be obstructed. Also, the talker may move from one place to the other, which could lead to a timevarying loop gain. This scheme can be undesirable in many instances. Thus, a public address system is suggested to have a capability that will automatically adjust with respect to its surroundings or to automatically compensate the presence of acoustic feedback problem. It is also emphasized that the importance of acoustic feedback canceller must be adaptive because the acoustic environment and conditions vary significantly and may be unexpected (Siqueira & Alwan, 2000).

Automatic feedback control methods are categorized into four classes, namely 1) Phase Modulation Methods, 2) Gain Reduction Methods, 3) Spatial Filtering Methods and 4) Room Modeling Methods. Such comparison of the four mentioned automatic feedback control method has not been reported earlier (Waterschoot & Moonen, 2011). This is presumably due to the fact that these methods attempt to solve different problems and hence different measures are being used in order to quantify the performance of each of these methods. Although the said automatic feedback control method deals with different problems but their objective is the same, which is to improve the capability of public address system by eliminating, reducing or avoid the occurrence of howling effect. Nevertheless, they differ in the way on how to eliminate, reduce or avoid the occurrence of howling effect. In general, automatic feedback control methods aim to eliminate, reduce or avoiding the occurrence of howling effect by controlling either one of the Nyquist criteria of stability.

Among the automatic feedback control methods, notch howling suppression (NHS) is the most popular. NHS method represents a traditional and robust solution; but its main disadvantage is, it is reactive and computationally complex (Rotaru, Stanciu, Ciochina, Albu, & Coanda, 2013). A reactive automatic feedback control will first allow the occurrence of howling before it is identified and eliminated. A more promising solution is to use adaptive feedback cancellation (AFC), which is based on acoustic feedback estimation (Rotaru *et al*., 2013). AFC is a type of room modeling method for automatic feedback control. The main objective in room modeling method is to obtain an equivalent model of the acoustic feedback path and attain the difference of the microphone signal and the output of the equivalent model of the acoustic feedback path. In this approach, acoustic feedback is expected to be further reduced or eliminated. The ways on how to obtain the acoustic feedback path can be either offline or online method. The difference between the two modeling methods depends on how the model is consequently applied for AFC. However, online methods gain some interest because acoustic feedback path may vary with time. This can be illustrated when the microphone is moved from one place to the other. Thus the distance between the microphone and the loudspeaker or the reverberation condition changes and so with the acoustic feedback path. As a result, the model obtained from offline room modeling method needs to be updated from time to time or when a change in acoustic feedback path has occurred. In online room modeling method, the acoustic feedback path is obtained from time to time without the needed human intervention. In a room modeling-based automatic feedback control, the equivalent acoustic feedback path model is used to estimate the feedback signal component in the microphone. The predicted signal is then subtracted to the microphone signal. Hence, the microphone signal is a feedback compensated signal using the estimate of the acoustic feedback path. If an accurate model of the acoustic feedback path is obtained, then the AFC is expected to have a complete elimination of the acoustic coupling (Waterschoot & Moonen, 2011). Cancellation of acoustic feedback will greatly depend on the acoustic feedback path estimate (Rotaru *et al*., 2013). Therefore, a very large maximum stable gain (MSG) can be achieved if the acoustic feedback path and acoustic feedback path estimate are identical. However, it has to be noted that in practice there is always a modeling error for many reasons, such as a slow adaptation speed and insufficient filter length (Ma *et al*., 2011).

Another room modeling approach is to acquire an equivalent inverse model of the acoustic feedback path. The inverse model can optimally equalize the microphone signal when inserted in the electroacoustic feedback path. This approach is also referred to as adaptive inverse filtering. Adaptive inverse filtering approach has gained little attention only in the context of AFC (Waterschoot & Moonen, 2011). Some works on adaptive inverse filtering for AFC can be found in some published works (Miyoshi & Kaneda, 1988; Elliott & Nelson, 1989; Ushiyama, Hirai, Tohyama, & Shimizu, 1994; Sarris, Jacobsen, & Cambourakis, 2004).

One of the popular room modeling techniques is based on adaptive filtering. Adaptive filtering scheme is widely used in hearing aids and its application for acoustic feedback cancellation is extrapolated for use in sound reinforcement system. The popularity of the adaptive feedback cancellation can be attributed to its capability that it can provide a higher MSG than other technique (Chi, Gao, Soli, & Alwan, 2003).

The implementation of an adaptive filter-based adaptive feedback cancellation for acoustic feedback control is similar to the well-known acoustic echo cancellation (AEC) approach (Waterschoot & Moonen, 2011). The adaptive filter is used to model and track the acoustic feedback path while the equivalent model will be used to produce a feedback compensated electroacoustic signal. However, in AFC, the disturbance signal, and the adaptive filter input signal are correlated. The correlation of the disturbance signal and the adaptive filter input signal will lead to a biased estimate of the acoustic feedback path (Gou, 2012). Furthermore, one of the biggest problems in using the adaptive filter for acoustic feedback cancellation is the biased estimation of the acoustic feedback path (Gou, 2012). The effect of a biased estimate will lead to a large modeling error and a cancellation of the desired signal (Ma *et al*., 2011). The resulting effect of the biased estimate is that the adaptive filter does not only predict and cancel the feedback component in the microphone signal but also part of the source signal (Waterschoot  $\&$ Moonen, 2011). As a consequence, the feedback compensated signal in the electroacoustic path is a distorted estimate of the microphone signal. Hence, it is important to eliminate the correlation of the disturbance signal and the adaptive filter input signal. The concept of AFC is quite simple and similar to AEC, however, its realization is not straightforward (Waterschoot & Moonen, 2011). This is because aside from the main objective of the AFC which is to eliminate or prevent the occurrence of howling, the AFC must also have the capability to eliminate the correlation of the disturbance signal and the adaptive filter input signal. Thus, the only way to avoid a biased estimate of the acoustic feedback path is to decorrelate the disturbance signal and the adaptive filter input signal. The work of Waterschoot & Moonen (2011) discussed the popular methods on how to decorrelate the disturbance signal and the adaptive filter input signal. The popular decorrelation methods for AFC are: 1) noise injection, 2) nonlinear filtering, and 3) unit delay.

The decorrelating method used in this paper is a unit delay. The delay is inserted in the electroacoustic path before the electroacoustic signal is being inputted to adaptive filter input. It is indicated in (Bustamante, Worrall, & Williamson, 1989), that a delay of 1 ms placed in the cancellation path is sufficient for decorrelating speech signals. Also, a delay of 2 ms was introduced in order to reduce the correlation of the speech signal (Berdahl *et al*., 2005). However, it is important to note that the delay in the forward path is particularly useful for near-end signals that have an autocorrelation function that

decays rapidly such as voiceless speech signals, provided that the delay value  $D$  is chosen accordingly (Waterschoot & Moonen, 2011). The selection of the delay  $D$  should be chosen to be large enough such that the speech would largely be uncorrelated with itself, while the delay  $D$  should be small enough such that the howling could be canceled before it grew too much in magnitude (Berdahl *et al*., 2005). Moreover, by making correspond to the same delay imposed by the cascade of the ADC and DAC, the adaptive filter can be made to converge to a transfer function that models the transfer function of the cascade of the DAC, speaker, room, microphone and ADC. Meanwhile, the resulting correlation effect is similar to having an adaptive filter preceded by a processing delay and when a processing delay is inserted in the electroacoustic forward path (Ortega, Lleida, & Masgrau, 2005). But it was emphasized in (Waterschoot & Moonen, 2011) that the delay length in the adaptive filtering circuit should not exceed the initial delay in the acoustic feedback path impulse response. On the contrary, in any given room's impulse response that can last up to few seconds, one might imagine the need for adaptive FIR filters implementing tens of thousands of adaptive coefficients (Berdahl *et al*., 2005). But because the talker might move quickly, an adaptive filter should adapt quickly. However, the excess mean square error (MSE) of the least mean square (LMS) adaptive algorithm increases with the number of filter taps when the rate of adoption is held constant. This implies that very long filters should have a convergence problem. Adaptive feedback cancellation has been accounted in real time implementation, however, one of its main disadvantages is its computational complexity. Computational complexity in AFC can be attributed to: 1) a very high adaptive filter order is required because the acoustic feedback path is being modeled by its impulse response and 2) the impulse response is densely sampled, which requires many coefficients and a large number of iterations has to be performed per second. Thus, it has to be noted that high adaptive filter order implementation has to consider: 1) the number of multiplication or DSP blocks to be used and 2) latency of the hardware where the adaptive filter will be implemented.

The design of AFC will be focused on preventing the magnitude condition of Nyquist criterion for instability from being met. The AFC will use an adaptive filter to model the acoustic feedback path through an adaptive algorithm. The equivalent model of the acoustic feedback path will be used to eliminate the acoustic feedback in the electroacoustic forward path. The concepts on adaptive filtering and adaptive feedback cancellation will be discussed in subsequent sections.

The adaptive filter based on LMS adaptive algorithm will serve as a foundation for designing an AFC in this study. The adaptive filter shown in Figure 2 will model the acoustic feedback path. It has a Discrete-Time Finite Impulse Response (DTFIR) structure that is based on tapped-delay-line and a set of N length of adjustable gain  $w_0$ through  $w_{N-1}$ .



Figure 2. Adaptive filter structure of the acoustic feedback path.

The adaptive filter output  $y(k)$  is the sum of the delayed and scaled inputs that is described in Eq. (4). It can also be simplified in a form as described in Eq. (5) as the dot product of the two vectors, namely, the weight vector in Eq. (6) and an input vector in Eq. (7).

$$
y(k) = \sum_{i=0}^{N-1} w_i x(k - i)
$$
 (4)

$$
y(k) = WT X = XT W
$$
 (5)

$$
W = \begin{bmatrix} W_1 & W_2 & \cdots & W_{N-1} \end{bmatrix}^T \tag{6}
$$

$$
X = \begin{bmatrix} x_1 & x_2 & \cdots & x_{N-1} \end{bmatrix}^T \tag{7}
$$

The error signal  $e(k)$  is derived from the difference of the desired signal  $d(k)$  and adaptive filter output signal  $y(k)$ .

$$
e(k) = d(k) - y(k) \tag{8}
$$

The error signal in Eq. (8) can also be written in a form as illustrated in Eq. (9) and Eq.  $(10)$ , with the use of Eq.  $(4)$  and Eq.  $(5)$ .

$$
e(k) = d(k) - \sum_{i=0}^{N-1} w_i x(k - i)
$$
 (9)

$$
e(k) = d(k) - WT X
$$
 (10)

The objective of the adaptive algorithm is to find the best impulse response vector W, which is also called the desired choice of weight vector and it is denoted as  $W^0$  in this paper. The best impulse response vector is the choice of the weight vector that makes the summed square difference between  $d(k)$  and  $y(k)$  as small as possible. The sum of the square difference is the MSE or sometimes called as the performance function which is denoted as  $\xi$  in this paper.

$$
\xi = \sum_{k=0}^{N-1} [d(k) - y(k)]^2 \tag{11}
$$

$$
\xi = \sum_{k=0}^{N-1} [d(k) - \sum_{i=0}^{N-1} w_i x(k-i)]^2
$$
\n(12)

For a given sequence of input vector  $X(k)$  and scalar  $d(k)$ , the MSE  $\xi$  is said to be a function of a weight vector W only. Therefore, the MSE  $\xi(W)$  is the measure of how well the weight vector  $W$  executes as a filter's impulse response that produces an output  $y(k)$ . It also describes the difference between desired signal  $d(k)$  and system's output signal  $y(k)$ .

The choice of W that minimizes the MSE  $\xi(W)$  is the value that produces an adaptive filter's output  $y(k)$ . To find the  $W^0$  that causes the MSE  $\xi(W)$  to be at a minimum level, first, expand Eq.  $(11)$  with the use of Eq.  $(5)$ , which defines the adaptive filter output  $y(k)$ .

$$
\xi(W) = \sum_{k=0}^{N-1} [d(k)]^2 - 2W^T \sum_{k=0}^{N-1} d(k) X(k) + W^T [\sum_{k=0}^{N-1} X(k) X(k)^T] W \tag{13}
$$

To further simplify Eq. (13) the following terms are defined:

$$
D = \sum_{k=0}^{N-1} [d(k)]^2
$$
 (14)

$$
P = \sum_{k=0}^{N-1} d(k) X(k) = \begin{bmatrix} d(k) x_{0,k} \\ d(k) x_{1,k} \\ \vdots \\ d(k) x_{N-1,k} \end{bmatrix}
$$
(15)

$$
R = \sum_{k=0}^{N-1} X(k)X(k)^{T} = \begin{bmatrix} x_{0,k}x_{0,k} & \cdots & x_{0,k}x_{N-1,k} \\ \vdots & \ddots & \vdots \\ x_{N-1,k}x_{0,k} & \cdots & x_{N-1,k}x_{N-1,k} \end{bmatrix}
$$
(16)

From Eq. (14) to Eq. (16), the term  $D$  is a scalar constant because it does not have a vector W as its coefficient. The P is a cross-correlation of the desired signal  $d(k)$  and input signal  $x(k)$ , which describes the correlation properties of  $d(k)$  and  $x(k)$  over an Npoint choice of shifts and R is the autocorrelation of the input signal  $x(k)$ . The diagonal elements of R measure the energy of the signal  $x(k)$  and also known as the eigenvalues of the autocorrelation of *.* 

In order to find the minimum Mean Square Error (MSE) we take the gradient of the MSE  $\xi(W)$  in Eq. (10) with respect to the elements of W where the weight vector W is set at its optimum value  $W^0$ . The gradient of the MSE  $\xi(W)$  is set to zero. The gradient of MSE  $\xi(W)$  is the same as forming a vector of partial derivatives of the MSE  $\xi(W)$  with respect to the impulse response coefficients.

$$
\nabla_{\mathbf{w}}\xi(W) = -2P + 2RW\tag{17}
$$

$$
W^0 = R^{-1}P \tag{18}
$$

On the other hand, it is very hard to implement the inversion of the autocorrelation of the input signal  $x(k)$  that is denoted as  $R^{-1}$  (Treichler, Johnson, & Larimore, 2001): 1) the inversion of  $R$  may not be possible and 2) if  $R$  were theoretically invertible, the

numerical precision required to invert  $R$  properly may be beyond the capability of the hardware or computer used in implementing the adaptive filter. For this reason, in order to find the optimum weight vector  $W^0$ , we have to search for the function of the MSE  $\xi(W)$  from the initial condition to the point where the MSE  $\xi(W)$  is at its minimum value and this can be done with the use of the steepest descent method. It is also important to note that the MSE  $\xi(W)$  in Eq. (13) is a quadratic function of the weight vector  $W$  and has a bowl-shaped surface.

The method of steepest descent approximates the weight vector proportional to the negative of the gradient vector. In steepest descent, the filter coefficient of  $w(k)$  has a large value of MSE  $\xi(W)$  when the weight vector is not equal to the best weight vector. The goal in here is to decrease the MSE  $\xi(W)$  and this can be done by moving  $w(k)$ towards the optimum weight vector in an iterative process.

$$
W(k + 1) = W(k) - 0.5\mu \nabla_{w}\xi(W)
$$
 (19)

The gradient of  $\xi(W)$  with respect to the weight vector W can be estimated directly from the desired signal  $d(k)$  and adaptive filter input signal  $x(k)$  by:

$$
\nabla_{\mathbf{w}} \xi(W) = -\nabla_{\mathbf{w}} [e^2(k)] \tag{20}
$$

$$
\nabla_{\mathbf{w}} \xi(W) = -2e(k)\nabla_{\mathbf{w}}[d(k) - W^T X(k)] \tag{21}
$$

$$
\nabla_{\mathbf{w}}\xi(\mathbf{W}) = 2e(k)X(k) \tag{22}
$$

By Eq. (21) and Eq. (22) a new weight vector can be approximated and can be written as shown in Eq. (23). The result is the LMS algorithm that is introduced by Widrow and Hoff in 1960.

$$
W(k + 1) = W(k) - \mu e(k)X(k)
$$
 (23)

The estimating equation of the weight vector as shown in Eq. (23) can be realized in a practical hardware without the need of squaring, averaging, inversion or differentiating and it is simpler to implement. Through iterative process and with the use of Eq. (4), Eq. (8) and Eq. (23), the weight vector  $W(k)$  will converge to the best weight vector at a right adaptation step size  $\mu$ .

The stability and convergence of the LMS algorithm depend largely on the adaptation step size  $\mu$ . This adaptation step size is a small constant that controls on how fast the algorithm will converge and approximate the desired or best weight vector. Using Eq. (17) in Eq. (19) the result is

$$
W(k + 1) = W(k) - \mu[-P + RW(k)]
$$
 (24)

$$
W(k + 1) = [I - \mu R]W(k) + \mu P
$$
 (25)

where *I* is an *N*-by-*N* identity matrix (Treichler, Johnson, & Larimore, 2001). As  $k$ approaches infinity,  $W(k)$  converges to the desired weight vector if the adaptation step size is small enough. In addition, it shows that in Eq. (25) using a small adaptation step size will lessen the effect of the autocorrelation  $R$  in approximating the weight vector. As a result, a more accurate approximation is expected because the weight vector is being varied but not that large to cause instability of the overall adaptive system. However, a smaller adaptation constant will result in a slower converging rate. Also even if the adaptation step size is small it does not mean that the stability of the system is assured, one should also consider the maximum magnitude of the input signal  $x(k)$  of the adaptive system. It shows that in Eq. (25), the deviation in updating the weight vector also depends on the input signal  $x(k)$  manifested in the autocorrelation R. Therefore in selecting an appropriate adaptation step size, one should also consider the maximum input signal. As a consequence, the selection for the adaptation step size in order to avoid instability is difficult and complex (Widrow & Stearns, 1985). The adaptive LMS algorithm is stable if and only if Eq. (26) is satisfied and can also be expressed in a form as shown in Eq. (27).

$$
|1 - 2\mu\lambda_{\text{max}}| < 1\tag{26}
$$

$$
0 < \mu < \frac{1}{\lambda_{\text{max}}} \tag{27}
$$

To effectively eliminate the howling effect in public address system, the acoustic feedback should be eliminated and prevent the Nyquist criterion for instability from being met. This will require an equivalent model of the acoustic feedback path, in which its output will be subtracted to signal from the microphone. However, the equivalent model of an acoustic feedback path is not directly available and sometimes may vary with time. Hence, an adaptive plant modeling using an adaptive filter was used in obtaining an equivalent mathematical model of acoustic feedback path as illustrated in Figure 3.



Figure 3. Block diagram of a single-channel public address system with adaptive feedback canceller (AFC).

The adaptive filter shown in Figure 3 will imitate the behavior of an acoustic feedback path in which it is considered to be unknown in this study. Both the adaptive filter and acoustic feedback path will be driven by a loudspeaker signal and the objective of the adaptive filter is to match its output signal to the output signal of the acoustic feedback path by simply adjusting the weight vector of the adaptive filter through an LMS algorithm. Hence, the adaptive filter will have an equivalent transfer function to the acoustic feedback path when the weight vector causes the MSE  $\xi(W)$  to be at its minimum (Widrow & Stearns, 1985). Upon convergence the structure and the parameter values may or may not be equal to those of the actual model of the acoustic feedback path but the input-output response relationship will be matched. In reality, the adaptive filter will not resemble an exact model of the unknown plant because of a minimum MSE  $\xi(W)$  manifested by the adaptive LMS algorithm. This can be illustrated by considering MSE  $\xi(W)$  in Eq. (13) with the use of Eq. (14) and Eq. (16), and then the MSE  $\xi(W)$  is

$$
\xi(W) = D - 2W^T P + W^T R W \tag{28}
$$

On the other hand, the equation that describes the desired weight vector as shown in Eq. (13) that causes the MSE  $\xi(W)$  to be at its minimum value can also be written as shown in Eq. (29).

$$
P = RW^0 \tag{29}
$$

Therefore the Mean Square Error (MSE)  $\xi(W)$  is at minimum when the weight vector has reached its optimum value and can be evaluated as

$$
\xi(W)_{\text{min}} = D - 2W^{0T}P + W^{0T}RW^0 \tag{30}
$$

$$
\xi(W)_{\text{min}} = D - W^{0T}P\tag{31}
$$

Thus, the minimum MSE  $\xi(W)$  described in Eq. (31) depends on the energy of the signal  $d(k)$ , the weight vector W and the correlation of signal  $x(k)$  and  $d(k)$ . In addition, random noise in the weight vector also adds misadjustment. If the weight vector is noise-free and converged to its desired setting, then the minimum MSE  $\xi(W)$  will be equal to zero. However, because of gradient noise present in the weight vector, the weight vector  $W$  will be noisy and on the average, it will be misadjusted from its desired or best setting and it will also exhibit a Brownian motion in the bottom of the MSE bowl (Widro & Walach, 2008). As a result, the average MSE  $\xi(W)$  is greater than the minimum MSE. Therefore, it is impossible to bring down the minimum MSE equal to zero (Widrow & Stearns, 1985). Lesser misadjustment can be achieved by letting the adaptive filter adapt slower, which can be done by having a smaller adoption step size (Widro & Walach, 2008). For this reason, the adaptive filter can only provide a close approximation of the acoustic feedback path and not the exact one.

Meanwhile, when an equivalent model of the acoustic feedback path is achieved by the adaptive filter, the overall transfer function of the public address system shown in Figure 3 will be equal to the equation shown in Eq. (32).

$$
\frac{x(z)}{v(z)} = \frac{G(z)}{1 - G(z)[F(z) - F'(z)]}
$$
(32)

The term  $F'(z)$  is the equivalent model of the acoustic feedback path  $F(z)$ . It is shown in Eq. (32) that acoustic feedback problem will be totally eliminated if the acoustic feedback path and its equivalent model are exactly matched. As a result, the MSG of the public address system is infinite or it would be impossible to experience a howling effect when  $F(z)$  and  $F'(z)$  exactly matches in any cases. However, as stated previously that it is impossible to obtain an exact match of  $F(z)$  and  $F'(z)$ , then we expect that the overall performance will be improved and the MSG will be bounded to a certain level higher to the system without AFC. In this case, in order for the public address system with adaptive feedback cancellation to remain stable and prevent the occurrence of howling effect, then it must avoid satisfying the Nyquist criterion for instability. The public address system must remain in the condition as shown in Eq. (33) and Eq. (34) in order to avoid howling effect.

$$
|G(z)[F(z) - F'(z)]| \ge 1
$$
\n(33)

$$
\angle G(z)[F(z) - F'(z)] = n(2\pi), \quad n = \dots, -2, -1, 0, 1, 2, \dots
$$
 (34)

On the other hand, considering Figure 1-3, the microphone signal  $d(z)$ , which is mathematically described as shown in Eq. (35).

$$
d(z) = v(z) + f(z) \tag{35}
$$

If Eq. (35) is used to express the weight vector described in Eq. (18) with the microphone signal  $d(z)$  as the desired signal of the adaptive filter, then the resulting equation is shown in Eq. (37).

$$
W^{0} = [X(z)X(z)^{T}]^{-1}[X(z)d(z)] \tag{36}
$$

$$
W^{0} = [X(z)X(z)^{T}]^{-1}[X(z)[\nu(z) + f(z)]] \qquad (37)
$$

Simplifying Eq. (37) and letting the feedback signal  $f(z)$  be equated with the ideal weight vector  $W^0$  and loudspeaker signal  $X(z)$  as shown in Eq. (38).

$$
f(z) = X(z)^T W^0 \tag{38}
$$

Using Eq. (38), the ideal weight vector in Eq. (37) can be expressed as

$$
W^{0} = [X(z)X(z)^{T}]^{-1}[X(z)[\nu(z) + X(z)^{T}W^{0}]]
$$
\n(39)

$$
W^{0} = W^{0} + [X(z)X(z)^{T}]^{-1}[X(z)v(z)] \qquad (40)
$$

The result in simplifying Eq. (39) is shown in Eq. (40). Eq. (40) shows that the ideal weight vector is biased. The biases in the weight vector will lead to a distorted feedbackcompensated electroacoustic signal. The biased problem shown in Eq. (40) in adaptive feedback canceller can be resolved if the term  $X(z)v(z)$  is equal to zero. This can be done by applying a decorrelating technique on either of the electroacoustic forward path or in the adaptive filter. In this study, a unit delay decorrelating technique was used.

$$
X(z) = G(z)[e(z - d)] \tag{41}
$$

The input signal of the adaptive filter in AFC with delay as decorrelation technique is shown in Eq. (41). The implementation of a delay based decorrelation technique for AFC may be simple and straightforward but the delay  $d$  should be chosen accordingly. The delay  $d$  should be chosen not so large that it could not cancel the growing feedback signal and not so small that it is unable to decorrelate the feedback signal and the input signal.

## **2. METHODOLOGY**

In this study, a Finite Impulse Response (FIR) adaptive filter was used as an automatic acoustic feedback controller. It is being implemented in real-time application using Altera DE1-SoC FPGA development board with an embedded audio codec.

The AFC was designed in Altera DE1-SoC FPGA development board. The acoustic signal from the talker and the acoustic feedback signal will be picked up by a microphone. In the microphone, the acoustic signal will be converted to an electrical signal then it will be fed to the Altera DE1-SoC FPGA development board for processing, enhancement and, more importantly, to eliminate the acoustic feedback. Then, the output signal of the Altera DE1-SoC FPGA development board is a feedbackcompensated electroacoustic signal that will be amplified by a power amplifier and converted back to acoustic signal using a loudspeaker.

In Figure 4, the Altera DE1-SoC FPGA development board will receive the electrical signal from the microphone through its embedded audio codec ADC. The audio codec ADC will be responsible for converting an analog electrical signal from the microphone to digital format. Furthermore, the audio codec will pre-amplify the analog signal in order to restore its integrity. Then, it will be sampled at a determined sampling rate and quantized. After quantization, it will be coded to 16 data bits. The audio codec's operational settings are to be configured by addressing its internal register using I2C protocol. The module that will address the internal register of the audio codec through I2C protocol was designed in FPGA. Meanwhile, the data from the audio codec ADC is in serial form and it will be received by a digital audio interface Rx module. The digital audio interface Rx is responsible for converting a serial data to parallel form. The data from the digital audio interface Rx module represents the acoustic signals that have been picked up by the microphone. In order to eliminate the acoustic feedback signal that has been picked up by the microphone, the data from the digital audio interface Rx module will be sent to a summer block. In the summer block, the acoustic feedback signal will be eliminated by subtracting the signal from the digital audio interface Rx module with the adaptive filter's output signal. The resulting difference is a feedback-compensated signal and it will be delayed by  $D$  samples before it will be sent to digital audio interface  $Tx$  for parallel to serial conversion. Meanwhile, the adaptive filter will take its input from the delayed feedback-compensated signal.



Figure 4. Block diagram of a field programmable (FPGA)-based adaptive feedback canceller (AFC).

The adaptive filter through its adaptive algorithm will obtain an equivalent model of the acoustic feedback path so that the adaptive filter's output signal will be an exact match of the acoustic feedback signal being picked up by the microphone. Its implementation is shown in Figure 5. The equivalent model of the acoustic feedback path can be obtained by varying the weights of the adaptive filter through its adaptive algorithm. The adaptive algorithm will vary the weights of the adaptive filter based on the feedback-compensated signal and the delayed feedback-compensated signal. The objective of the adaptive algorithm is to match the adaptive filter's output signal and the acoustic feedback signal. When the best match is achieved, the signal sent to the digital audio interface Tx module is a feedback-compensated signal. The digital audio interface Tx module will convert the parallel form of the acoustic feedback compensated signal to a serial form for digital to analog conversion. The analog signal from the DAC will be amplified by a power amplifier and convert it back to the acoustic signal by a loudspeaker. On the other hand, the delay block in the acoustic feedback controller is used to avoid a biased solution of the adaptive algorithm, which is due to the correlation between the acoustic feedback signal and the adaptive filter's input signal.

The performance of the public address system was firstly evaluated without acoustic feedback canceller. The microphone and the loud speaker were placed at a distance of about three meters. The microphone was connected to the microphone input of the Altera DE1-SoC development board. Next, the Altera DE1-SoC development board is connected to a power amplifier, which is then connected to the loud speaker. First, the Altera DE1-SoC FPGA's Cyclone V was configured to directly connect the audio input and output without any attenuation or amplification. The power amplifier's gain was slowly increased from the minimum of -73 dB to a gain in which the acoustic feedback will occur. Then, the acoustic environment will be observed using a spectrum analyzer and spectrograph. In this way, the characteristic of acoustic feedback can be visualized and analyzed with and without music signal being applied. Then, the AFC was employed and the evaluation was redone at different adaptation constant. The implementation of the adaptive filter in Altera Cyclone V with 20 filter taps is shown in Figure 5. It is composed of D flip-flops, multipliers and adders. The arithmetic operation is performed using a 16-bits fixed-point data format. Meanwhile, the decorrelation  $D$  is set to 20 ms. It is assumed that a 20 ms delay for decorrelation is enough to decorrelate  $X(z)$  and  $v(z)$  and at the same time effectively eliminate the acoustic feedback that might occur.

A music signal was also applied on the microphone of the public address system without AFC. The acoustic environment of the room was then observed using a spectrograph and slowly increased the gain of the power amplifier from -73 dB to which an audible howling is noticeable. The results of the evaluation shows the characteristic of an amplified music signal with and without noticeable howling. The AFC was then employed at different adaptation constant and the same music signal was applied at the microphone of the public address system. The adaptive filter was given a considerable time to settle at a weight vector solution at a power amplifier's gain of 3 dB lower to which howling was observed without AFC. Then the gain of the power amplifier was slowly increased to a maximum power amplifier's gain where there is no howling. Then, the acoustic environment will be observed by a spectrograph and the frequency spectrum will be compared. The results of the evaluation will show any presence of howling with the amplified music signal. The evaluation by comparing the frequency spectrum of the observed acoustic at different adaptation constant will also show what will be the effect of the adaptation constant on the achievable amplification of the public address system with AFC.



Figure 5. FPGA implementation of  $N$ -taps adaptive filter.

## **3. RESULTS AND DISCUSSION**

Without AFC, the gain of the power amplifier is slowly increased from -73 dB to which howling will occur. As the gain of the power amplifier is increased from -73 dB, howling was observed at a power amplifier's gain of -6 dB. The acoustic environment of the room was then observed using a spectrum analyzer and a spectrograph. The spectrum at a time instant is shown in Figure 6 and the spectrograph is shown in Figure 7.

Figure 6 shows that there are narrow frequency bands that have higher magnitude. However, there is a narrow frequency band that dominates among the rest. This frequency component measured at 133 Hz is shown in Figure 6 that dominates the rest is said to be the audible monotone which is the howling signal. When the audible monotone is observed using spectrograph, Figure 7 shows that the dominant frequency component remains dominant throughout the existence of howling. This shows that the frequency component of the howling signal remains stationary throughout the existence of howling. This also shows that if the adaptive filter is employed to eliminate the howling signal using the configuration shown in Figure 3, the adaptive filter must detect the existence of howling signal and be able to quickly generate a signal similar to howling signal in magnitude and frequency. Thus, as a result, the stationary acoustic feedback signal and the adaptive filter output signal cancel one another. In consideration also with the capability of the adaptive filter that acts as an acoustic feedback canceller, the adaptive filter can detect howling signal and be able to produce an output of the same characteristics with the howling signal if it remains stationary throughout its existence and the acoustic feedback path does not change. However, the adaptive filter must quickly produce an output of the same characteristic with the howling signal before the overall system will exhibit an unstable behavior.



Figure 6. Frequency spectrum of the observed acoustic signal of howling at a time the microphone and the loudspeaker were placed at a distance of about three meters within a room.



Figure 7. Spectrograph of the observed acoustic signal of howling.

The gain of the power amplifier was then reduced to -73 dB and the adaptive filter was employed with different adaptation constant. Then, the gain of the power amplifier was slowly increased until howling occurs. As the gain of the power amplifier is slowly increased, it was observed that when the power amplifier's gain reaches -6 dB, the audible monotone cannot be heard anymore. The audible monotone was then observed again above -6 dB of the power amplifier's gain. The difference between the gains of the power amplifier with and without AFC is said to be the improvement of the maximum stable gain (ΔMSG) of the public address system. The ΔMSG of the public address system at different adaptation constant of the adaptive filter is shown in Table 1.

Table 1 shows that the higher the adaptation constant the higher the ΔMSG. This shows that the AFC quickly eliminates the howling signal with higher adaptation constants. As a result, AFC with higher adaptation constant leads to a higher ΔMSG. However, adaptive filter with higher adaptation constant approximates the desired signal less accurate and less precise compared to the adaptive filter with smaller adaptation constant. Furthermore, if an acoustic signal in a form of music is applied at the microphone of the public address system with AFC, the AFC should only eliminate the howling signal and should not affect the music signal. Thus, the AFC should not only quickly eliminate the howling signal but should also distinguish the difference of the howling signal and the music signal.

Adaptation Constant $(\mu)$	∆MSG
0.0001	5 dB
0.001	5 dB
0.01	6 dB
	6 dB

Table 1. Improvement of maximum stable gain (MSG) of the public address system with adaptive feedback canceller (AFC).

An acoustic signal in a form of music was applied at the microphone of the public address system without AFC. The power amplifier's gain was then increased to -6 dB and the acoustic environment was observed using spectrograph. Also at -6 dB of the power amplifier's gain, an audible monotone aside from the music signal is observable. The spectrograph of the acoustic signal at -6 dB of the power amplifier's gain is shown in Figure 8. Figure 8 shows that a howling signal of 133 Hz is present in the amplified music signal. It also shows that the magnitude of the 133 Hz howling signal also dominates the rest of the frequency component and its characteristics remain the same throughout its existence. Thus, the 133 Hz howling signal remains stationary even if there is an acoustic signal in a form of music applied at the microphone of the public address system. Then the gain of the power amplifier was reduced to -9 dB in which the howling cannot be observed anymore and the music was replayed. Then the acoustic environment was observed. The spectrograph of the acoustic signal with -9 dB of the power amplifier is shown in Figure 9. In Figure 9, it can be observed that dominant frequencies do not remain fixed over a long period.



Figure 8. Spectrograph of music signal without AFC at -6 dB of power amplifier's gain.



Figure 9. Spectrograph of music signal without AFC at -9 dB of power amplifier's gain.

The power amplifier was first set to -9 dB then the music signal was applied. It is also presumed that the adaptive filter was now able to settle at the desired weight vector after 20 seconds. Then 20 seconds after the music was applied, the power amplifier's gain was slowly increased at a rate of almost 1 dB/s. After that, howling was observed when the gain of the power amplifier reached 0 dB. The process of validation of the public address system with AFC was redone and the gain of the power amplifier was slowly increased from -9 dB to -1 dB. The acoustic signal with AFC using an adaptation constant of 0.0001 and 0.1 was observed and the spectrograph is shown in Figure 10.



Figure 10. Spectrograph of music signal with AFC at 1 dB with (a)  $\mu = 0.0001$  and (b)  $\mu = 0.1$ 

It can be seen in Figure 10 that there is no observable dominant frequency that remains fixed over a long period. This shows that the howling component observed in Figure 8 with the power amplifier's gain at -6 dB was eliminated when AFC was used. However, the difference on the result of the two adaptation constants was not noticeable as shown in Figure 10.

To further show that the music signal was amplified with the amplifier's gain of -1 dB without any observable howling signal, the frequency spectrum of the signal in Figure 10a and Figure 10b was compared to the frequency spectrum of Figure 9 in which all three signals have no observable howling. The results of the comparison in frequency spectrum are shown in Figure 11. In Figure 11, results show that the public address system with AFC was able to achieve an amplification greater than the public address system without AFC. However, comparing the public address system with AFC and different adaptation constant results in Figure 11 shows that at the significant frequency band, the AFC with smaller adaptation constant achieved amplification greater than the AFC with larger adaptation constant. This is because the adaptive filter with higher adaptation constant approximates the howling signal less accurate and less precise compared to the adaptive with smaller adaptation constant. Furthermore, with the music signal applied on the microphone, the AFC with higher adaptation constant, because it quickly reacts, it does not only attenuate the howling signal but also attenuates the music signal being applied at the microphone. This shows why the AFC with smaller adaptation constant achieved larger amplification compared to AFC with large adaptation constant.



Figure 11. Frequency spectrum comparison of music signal without and with AFC at -9 dB and -1 dB, respectively.

#### **4. CONCLUSIONS**

In this paper, an adaptive filter using LMS adaptive algorithm was employed as an acoustic feedback canceller for public address system. The public address system with AFC was evaluated in real time environment using Altera DE1-SoC. The adaptive filter with 20 filter taps and an LMS adaptive algorithm were implemented in an Altera Cyclone V FPGA with the objective to classify and eliminate howling signal when it occurs.

Results show that the adaptive filter using LMS adaptive algorithm successfully classifies and automatically eliminates the howling signal. The result also shows that with the incorporation of adaptive filter as AFC allows the public address systems to have an additional 5 dB to which it still exhibits a stable behaviour. In addition, the acoustic signal of public address system at a stable gain shows that the one with AFC is greater than 5 dB on average and a maximum of 12 dB compared to public address system without AFC. However, the effect of the adaptation constant results shows that, with the music signal applied on the microphone, the AFC with higher adaptation constant, because it quickly reacts, it does not only attenuate the howling signal but also attenuates the music signal being applied at the microphone. This shows why the AFC with smaller adaptation constant achieved larger amplification compared to AFC with large adaptation constant.

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